

SECOND PUC PREPARATORY EXAMINATION - 2020

Time : 3 Hrs. 15 Mins.

MATHEMATICS (35)

Max Marks : 100

No. of Pages : 03

Total No. of Ques. : 50

PART - A

10x1=100

I Answer all the following questions :

- 1) Give an example to show that $*$: $N \times N \rightarrow N$ given by $*(a, b) = a - b$ is not a binary operation.
- 2) Write the domain of $f(x) = \sec^{-1} x$
- 3) Find the number of all possible matrices of order 3×3 with each entry 0 or 1.
- 4) Define adjoint of a matrix.
- 5) Write the points of discontinuity for the function $f(x) = [x]$, $-3 < x < 3$.
- 6) Write the anti-derivative of e^{2x} with respect to x .
- 7) Find the angle between the two vectors \vec{a} and \vec{b} such that $|\vec{a}| = 1$, $|\vec{b}| = 1$ and $\vec{a} \cdot \vec{b} = 1$.

- 8) Find the direction ratio of the line $\frac{x-1}{2} = 3y = \frac{2z+3}{4}$

9) Define constraints in LPP.

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	-0.1	0.3

Given is not a probability distribution why?

PART - B

10x2=200

II Answer any TEN of the following questions :

- 11) Prove that the greatest integer function, $f: R \rightarrow R$, defined by $f(x) = [x]$, where $[x]$ indicates the greatest integer not greater than x , is neither one-one nor onto.
- 12) Write the function $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $0 < x < \pi$ in the simplest form.
- 13) Prove that $2 \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{24}{7}\right)$.
- 14) If each element of a row is expressed as sum of two elements then verify for a third order determinant that the determinant can be expressed as sum of two determinants.
- 15) Find $\frac{dy}{dx}$, if $\sin^2 x + \cos^2 y = k$, where k is constant.
- 16) If $y = \tan^{-1} \frac{\sin x}{1 + \cos x}$ then prove that $\frac{dy}{dx} = \frac{1}{2}$.
- 17) Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.
- 18) Evaluate $\int e^x \sec x (1 + \tan x) dx$.
- 19) Evaluate $\int \log x dx$
- 20) Prove that the differential equation $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$ is a homogeneous differential equation of degree 0.
- 21) If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.

(P.T.O.)

22) Find the area of parallelogram whose adjacent sides are given by the vectors

$$\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k} \text{ \& } \vec{b} = \hat{i} - \hat{j} + \hat{k}$$

23) Find the distance of point (2, 3, -5), from the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} - 2\hat{k}) = 9$

24) A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F is the event 'the number appearing is even', then find whether E and F are independent.

PART - C

III Answer any TEN of the following questions : 10x3=30

25) Show that the relation R in the set of real numbers R defined as $R = \{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

26) Simplify : $\tan^{-1} \left[\frac{2 \cos x - 3 \sin x}{3 \cos x + 2 \sin x} \right]$ where $\frac{2}{3} \tan x > -1$.

27) If A & B are square matrices of the same order, then show that $(AB)^{-1} = B^{-1} A^{-1}$

28) Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ w.r.t. x

29) Verify mean value theorem for the function $f(x) = x^2 - 4x - 3$ in the interval [1, 4].

30) Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

31) Evaluate $\int \frac{dx}{x(x^n + 1)}$

32) Evaluate $\int_1^5 (x+1) dx$ as a limit of sum.

33) Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and X-axis in the I quadrant.

34) Form the differential equation of the family of circles touching the y-axis at origin.

35) Find the unit vector perpendicular to the vectors $\hat{a} + \hat{b}$ and $\hat{a} - \hat{b}$ where $\hat{a} = \hat{i} + \hat{j} + \hat{k}$ and $\hat{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

36) For any three vectors \vec{a}, \vec{b} and \vec{c} prove that $\begin{vmatrix} \vec{a} + \vec{b} & \vec{b} + \vec{c} & \vec{c} + \vec{a} \end{vmatrix} = 2 \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \end{vmatrix}$

37) Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

38) Two groups are competing for the position on the board of directors of a corporation. The probability of I and II groups will win are 0.6 and 0.4 respectively. Further, if I group wins, the probability of introducing a new product is 0.7 and corresponding probability is 0.3, if the II group wins. Find the probability that new product introduced was by the II group.

PART - D

IV Answer any SIX of the following questions : 6x5=30

39) Prove that the function, $f : N \rightarrow Y$ defined by $f(x) = x^2$, where

$Y = \{y : y = x^2, x \in N\}$ is invertible. Also write the inverse of $f(x)$.

40) If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B = [1 \ 3 \ -6]$ verify that $(AB)' = B'A'$.

(P.T.O.)

