Jain College, Jayanagar
II PUC Mock Paper I - Jan 2019
Subject: II PUC Mathematics (35)

Duration: 3 hours 15 minutes **Max. Marks: 100**

PART-A

I. Answer all the TEN questions: **10X1=10**

1. Show that the relation R in the set {1,2,3} given by R = {(1,2) (2,1)} is not transitive.
2. Write the domain of f(x) = $\cos^{-1}x$.
3. Write the condition for the matrix A = $[a_{ij}]_{m \times n}$ to be square matrix.
4. If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find $|2A|$.
5. Find $\frac{dy}{dx}$, if y = sin (x^2)
6. Find $\int Sec(\sec x + \tan x)\,dx$
7. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.
8. If a line has the direction ratios -18, 12, -4 then what are its direction cosines?
9. Define objective function in linear programming problem.
10. Compute P(E/F), if P(F) = 0.5 and P(E∩F) = 0.32.

PART-B

II. Answer any TEN questions:
10X2=20

11. Show that f : A →B & g: B →C are one – one, then g•f : A→C is also one-one.
12. Prove that, $Sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x, \frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$
13. Prove that Sin ($\tan^{-1}x$) = $\frac{x}{\sqrt{1+x^2}}$
14. Prove that the value of the determinant of order 3 remains unchanged if its rows and columns are interchanged. (1)
15. If $\sqrt{x} + \sqrt{y} = 10$ show that $\frac{dy}{dx} + \sqrt{\frac{y}{x}} = 0$.
16. Find $\frac{dy}{dx}$ if $y = Cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right], 0 < x < \pi.$
17. Find he slope of the tangent to curve y = x^3 –x +1 at the point whose x – coordinate is 2.
18. Evaluate $\int \cos^{-1}(\sin x)dx$.
19. Evaluate : $\int \frac{2x \tan^{-1}x^2}{(1+x^4)}dx$.
20. Verify that the given function y = Ax is a solution of xy` = y (x≠0).
21. Find the area of the parallelogram whose diagonals are represented by the vectors. $\vec{d_1} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d_2} = \hat{i} - 3\hat{j} + 4\hat{k}$.
22. Find volume of parallelepiped whose coterminous edges are represented by the vectors $\vec{a} = \hat{i} - 2\hat{j} - 3\hat{k}$ and $\vec{d} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$.
23. Determine wether the given planes 7x+5y+6z+30=0 & 3x-y-10z+4=0 are parallel or perpendicular & in case they are neither.
24. A fair die is rolled. Consider the events E = {1,3,5} F={2 3} find P (E/F).

PART-C

III. Answer any TEN questions: 10X3=30

25. If R_1 and R_2 are two t equivalence relations on a set A, then show that $R_1 \cap R_2$ is also an equivalence relation.

26. Solve for x, if $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$

27. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ then find $(A+2B)'$

28. Find $\frac{dy}{dx}$ if $xy = e^{x-y}$.

29. Verify mean value theorem if $f(x) = x^3 - 5x^2 - 3x$ in the interval [1,3].

30. Find the two positive numbers x and y such that x+y = 60 and xy^3 is maximum.

31. Evaluate $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$

32. Evaluate $\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx$

33. Find the area bounded by the curve y = cos x between x = 0 and x = 2π.

34. Find the particular solution of differential equation $\log\left[\frac{dy}{dx}\right] = 3x+4y$, given y=0 when x=0.

35. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and each one of them being perpendicular to the sum of other two, find $|\vec{a}+\vec{b}+\vec{c}|$.

36. Show that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}, 3\hat{i}+9\hat{j}+4\hat{k}$ and $+4(-\hat{i}+\hat{j}+\hat{k})$ respectively are coplanar.

37. Find the distance of the point (-1, -5, -10) from the point of intersection of the line $\vec{r} = 2\hat{i}-\hat{j}+2\hat{k}+\lambda\,(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}.(\hat{i}-\hat{j}+\hat{k}) = 5$.

38. Three cards are drawn successively without replacement from pack of 52 well shuffled cards. What is the probability that first two cards are kings and the third card drawn is an ace?

PART-D

IV. Answer any SIX of the following: 6X5=30

39. Let f : N →R be a function defined as $f(x) = 4x^2 + 12x + 15$ show that f : N →S where 'S' is the range of f is invertible. Find the inverse of f.

40. For the matrices A and B, verify that $(AB)' = B'A'$, where A= $\begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ B= [1 5 7]

41. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

x – y + 2z=1
2y-3z=1
3x-2y+4z=2

42. If e^y (x+1) = 1. Prove that $\frac{dy}{dx} = e^{-y}$ hence prove that $\frac{d^2y}{dx^2} = \left[\frac{dy}{dx}\right]^2$.

43. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 cms?

44. Find the integral of $\sqrt{x^2 - a^2}$ with respect to x and hence evaluate $\int \sqrt{x^2 + 4x + 1}$ dx.

45. Find the area bounded by the curve x^2 =4y and the line x = 4y -2.

46. Find the general solution of the differential equation $x\frac{dy}{dx} + y - x + xy$ cot x = 0 (x≠0)

47. Derive the equation of plane passing through three non collinear points.

48. If a fair coin is tossed 10 times, find the probability of :
i) exactly six heads ii) at least six heads

PART-E

V. Answer any one of the following: 1X10=10

49. a) Prove that $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^{a} f(x)dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$ Hence evaluate $\int_0^{\pi} |\cos x|\,dx$ (6M)

b) By using the properties of determinants prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$ (4M)

50) Solve the following problem graphically:
a) Maximize and minimize z = x+2y subject to constraints. (6M)
x+2y ≥ 100
2x – y ≤ 0
2x+y ≤ 200
x, y ≥ 0,
b) Find all the points of discontinuity of the greatest integer function defined by
f(x) = [x] where (x) denotes the greatest integer less than or equal to x.
(4M)

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