JAIN COLLEGE

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Bangalore - 560 098

Date: ***SUBJECT: MATHEMATICS***

II PUC

MOCK 1

Timings Allowed: 3 Hrs 15 minutes. ***Total Marks:100***

Instructions: 1. The question paper has 5 parts A, B,C,D and E. Answer all parts.
2. Part A carries 10 marks, Part-B carries 20 marks, Part-C carries 30 marks, Part-D carries 30 marks and Part-E carries 10
3. Write the question number properly as indicated in the question paper

PART A

I Answer all **1X10=10**

1. Show that *:R→ $R, R\ is\ set\ of\ real\ numbers, given\ by\ (a * b) = a + 4b^2$ is a binary operation
2. Write the domain of f(x)=$\cos^{-1}$x
3. If A=$\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\end{bmatrix}$,B=$\begin{bmatrix}3 & -1 & 3\\-1 & 0 & 2\end{bmatrix}$ then find 2A-B
4. If A is an invertible matrix of order 2 and |A|=15. Then find det(A^{-1})
5. Differentiate sin(ax+b) w.r.t.x
6. Find the antiderivative of $x^2\left(1 - \frac{1}{x^2}\right)$ w.r.t.x
7. Find the value of x,y,z so that $\vec{a}$ =$x\hat{\imath} + 2\hat{\jmath} + z\hat{k}$,$\vec{b} = 2\hat{\imath} + y\hat{\jmath} + \hat{k}$ are equal
8. Show that the plane 2x-y+4z=5 and 5x-2.5y+10z =6 are parallel
9. Define corner points of LPP
10. An Urn contains 4 green and 2 black balls. Two balls are randomly selected . Let X represents the number of black balls. What are the possible values of X

Part B

II Answer any ten **2X10=20**

11. Prove that the greatest integer function f:R→ $R\ given\ by$ f(x)=[x],indicates greatest integer not > x, is neither 1-1 nor onto
12. Write the simplest form of $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$
13. Prove that $2\tan^{-1} 1/2 + \tan^{-1} 1/7 = \tan^{-1} 31/17$
14. Find k if A^2=KA-2I where A= $\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}$
15. Differentiate $(logx)^{cosx}$ w.r.t 'x'
16. Find $\frac{dy}{dx}$ if x=a($\theta - sin\theta$); y=a(1+cosθ)
17. Find the approximate change in surface area of a cube of side x mtrs caused by decreasing the side by1%
18. Evaluate $\int_0^2 \frac{6x+3}{x^2+4} dx$
19. Evaluate $\int \frac{1}{x(logx)^m} dx$
20. Find order and degree of $\left(\frac{d^4y}{dx^4}\right)^2 + \cos\left(\frac{dy}{dx}\right)$=0
21. Find the position vector of midpoint of vector joining p(2,3,4) ,Q(4,1,-2)
22. Find $|\vec{x}|$ if for a unit vector $\vec{a}$,($\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 12$

23. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other

24. A die is thrown .If E is the event "the number appearing is a multiple of 3 " and F is the event "the number appearing is even " then Find whether E and F are Independent.

PART C

III Answer any TEN **10X3=30**

25. Show that relation R in the set A={x : x∈ Z,0≤x ≤12} given by R={(a,b): |a-b| is a multiple of 4} is an equivalence relation

26. Show that the matrix $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ can be expressed as a sum of symmetric and skew symmetric matrix

27. Solve for 'x' $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$; x>0

28. If x= a $\cos^3 t$,y=a$\sin^3 t$,Show that $\left(\frac{dy}{dx}\right)=\sqrt[3]{\frac{y}{x}}$

29. Verify Rolles theorem $f(x)=x^2+2x-8, x\in[-4,2]$

30. Find two positive numbers whose sum is 15 ,and sum of whose squares is minimum

31. Evaluate $\int_{-1}^{1} 5x^4\sqrt{x^5+1}\,dx$

32. $Evaluate \int \frac{dx}{(x+1)(x+2)}$

33. *Find the area of the region bounded by* two parabolas $y=x^2$ and $y^2=x$

34. *Find the equation of the curve passing through point* (1,1)*whose d*ifferential equation is $xdy=(2x^2+1)dx$

35. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ Find the value of $\vec{a}.\vec{b}+\vec{c}.\vec{a}+\vec{b}.\vec{c}$

36. Show that four points A,B,C,D with position vectors $4\hat{\imath}+5\hat{\jmath}+\hat{k}$,$-(\hat{\jmath}+\hat{k})$,$(3\hat{\imath}+9\hat{\jmath}+\hat{k})$ *and* $-4(-\hat{\imath}+\hat{\jmath}+\hat{k})$*respectively* are coplanar

37. Find the equation of the plane passing through the line of intersection of planes $\vec{r}.(\hat{\imath}+\hat{\jmath}+\hat{k})=1$ and $\vec{r}.(2\hat{\imath}+3\hat{\jmath}+\hat{k})+4=0$ and parallel to x axis

38. Out of 38 students in a college it is known that 60%reside in hostel and 40% are day scholars. Previous year result report that 30% of all students who reside in hostel attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hosteller?

PART D

IV Answer any six **5X6=30**

39. Let f: N→R be a function defined as $f(x)=4x^2+12x+15$. Show that f: N→S where S is the range of f , is invertible. Find the inverse of f

40. If A=$\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ and B=$\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ $and\ C=\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$then compute A+B and B-C.Also verify that A+(B-C)=(A+B)-C

41. Solve the system of equations by Matrix method 2x+3y+3z=5; x-2y+z=-4 and 3x-y-2z=3

42. If $y=3e^{2x}+2e^{3x}$Prove that $y^{II}-5y^{I}+6y=0$

43. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi vertical angle is $\tan^{-1}(0.5)$.Water is poured into it at a constant rate.Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4m

44. Find the integral of $\frac{1}{\sqrt{a^2-x^2}}$ w.r.t 'x' .Hence Evaluate $\int \frac{1}{\sqrt{9-25x^2}}dx$
45. Using integration find the area of triangular region whose sides have equation y=2x+1,y=3x+1,x=4
46. Find the general solution of $\cos^2 x\frac{d^2y}{dx^2}+y=\tan x$
47. Derive equation of the line passing through 2 given points in space both in vector and normal form
48. A die is thrown 6 times. If getting an odd number is a success. What is the probability of (i)5 success (ii)at least 5 success

PART E

V Answer any one **1X10=10**

49. a) A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs. 17.50 per package of nuts and Rs.7.00 per package on bolts. How many packages of each should be produced each day so as to maximize profit.

b)If $f(x)=\begin{Bmatrix} 3ax+b & if\ x>1 \\ 11 & if\ x=1 \\ 5ax-2b & if\ x<1 \end{Bmatrix}$ is continuous at x=1. Find the values of a and b

50. a)Prove that $\int_{-a}^{a} f(x)dx=\begin{Bmatrix} 2\int_0^a f(x)dx & if\ f(x)\ is\ even\ function \\ 0 & if\ f(x)\ is\ odd\ function \end{Bmatrix}$ and hence evaluate

$$\int_{-\pi/2}^{\pi/2}(x^3+x\cos x+\tan^5 x+1)dx$$

b)Show that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$=(x-y)(y-z)(z-x)(xy+yz+zx)

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