



# JAIN COLLEGE

463/465, 18th Main Road, SS Royal, 80 Feet Road, Rajarajeshwari Nagar,

Bangalore - 560 098

Date:

II PUC

SUBJECT: Mathematics

MOCK - I

Timings Allowed: 3 Hrs 15 Minutes

Total Marks: 100

## PART A

### I. ANSWER ALL.

1X10=10

1. Define Objective function
2. Find principal value of  $\tan^{-1} \sqrt{3}$
3. A matrix has 8 elements, what are the possible orders it can have?
4. Find 'x' if A is a singular matrix  $A = \begin{pmatrix} 1 & x \\ x & 36 \end{pmatrix}$
5. Find derivative of  $\cos(x^2)$  w.r.t.'x'
6.  $(2x^2 + e^x)dx$
7. Write two different vectors having same direction
8. If a line makes angles  $90^\circ, 135^\circ, 45^\circ$  with x, y, z axes respectively, find their direction cosines
9. Define optimum solution in LPP
10. X represent difference between numbers of heads and number of tails when a coin is tossed 3 times. What are the possible values of x?

## PART B

### II ANSWER ANY TEN

2X10=20

11. Is \* defined on set {1,2,3,4,5} by  $a*b = \text{LCM of } a \text{ and } b$ , a binary operation?
12. Prove that  $2\tan^{-1}x = \tan^{-1} \frac{2x}{1-x^2}$  ;  $-1 < x < 1$
13. If  $\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \pi/4$ , Find x
14. Find area of triangle with vertices (1,0),(6,0),(4,3) using determinants
15. If  $x^{2/3} + y^{2/3} = a^{2/3}$  Find  $y'$
16. If  $y = (\cos^{-1}x)^x$  find  $y'$
17. Find local maximum and local minimum values of  $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$
18. Evaluate  $\int \log x \, dx$
19. Evaluate  $\int_0^{\pi/2} \sin 2x \, dx$
20. Prove that  $x^2y' = x^2 - 2y^2 + xy$  is a homogeneous D.E of degree 2
21. Show that the vectors  $2i - 3j + 4k$  and  $-4i + 6j - 8k$  are collinear
22. Find the area of triangle having the points A(1,1,1), B(1,2,3), C(2,3,1) as its vertices

23. Find equation of plane which passes through the intersection of the planes  $3x-y+2z-4=0$  and  $x+y+z-2=0$  and the point  $(2,2,1)$
24. An unbiased die is thrown twice. Let the event A be "odd number on the first throw", B the event "odd number on second throw" Are A and B independent?

### PART C

#### III ANSWER ANY TEN.

10X3=30

25. '\*' be a binary operation on N defined by  $a*b = \frac{ab}{4}$ . IS '\*' commutative and Associative?
26. Prove that  $\tan^{-1} x + \tan^{-1} \frac{2x}{1-x^2} = \tan^{-1} \frac{3x-x^3}{1-3x^2}$  ;  $|x| < 1/\sqrt{3}$
27. If A and B are square matrices of same order, Show that  $(AB)^{-1} = B^{-1}A^{-1}$
28. If  $y = \sin^{-1}x$  Show that  $(1-x^2)Y'' - xy' = 0$
29. If a function  $f(x)$  is differentiable at  $x=c$ , Prove that it is continuous at  $x=c$
30. A rectangular sheet of tin 45cmX24cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of square to be cut off so that the volume of the box is maximum?
31.  $\int_0^{\pi} \log 1 + \cos x \, dx$
32.  $\int e^{x^3} x^2 dx$
33. Find the smaller area enclosed by circle  $x^2 + y^2 = 2^2$  and the line  $x+y=2$
34. In a bank, principal increases continuously @r% per year. Find the value of r if Rs.100 double itself in 10 years ( $\log_e 2 = 0.6931$ )
35. Find a unit vector perpendicular to each of  $(a + b)$  and  $(a - b)$  Where  $a = i + j + k$  and  $b = i + 2j + 3k$
36. Show that the position vector of the point p which divide the line joining the points A and B having position vectors  $a$  and  $b$  internally in the ratio  $m:n$  is  $\frac{mb+na}{m+n}$
37. Find the distance between the lines  $r = (i+2j+k) + \alpha(i+2j+3k)$  and  $r = (2i - j - k) + \beta(2i+j+2k)$
38. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident are 0.01, 0.03 and 0.15 respectively. One of the insured person meets with an accident. What is the probability that he is a scooter driver?

### PART D

#### IV ANSWER ANY SIX.

6X5=30

39.  $f: R \rightarrow R, f(x) = 4x+3$  Show that f is invertible. Find the inverse of f
40. If  $A^T = \begin{pmatrix} -2 & 3 \\ 1 & 2 \end{pmatrix}$   $B = \begin{pmatrix} -1 & 0 \\ 1 & 2 \end{pmatrix}$  Verify  $(AB)^T = A^T B^T$
41. Solve by matrix method  $2x+3y+3z=5, x-2y+z=-4, 3x-y-2z=3$
42. If  $y = 3\cos(\log x) + \sin(\log x)$ , Show that  $x^2 y_2 + x y_1 + y = 0$
43. A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, @ 2cm/s. How fast is its height on the wall decreases when the foot of the ladder is 4m away from the wall?
44. Find  $\int \frac{1}{x^2-a^2} dx$  w.r.t.'x' hence evaluate  $\int \frac{1}{x^2-25} dx$

45. Find the area bounded by  $4x^2+4y^2=9$  , which is interior to the parabola  $x^2=4y$
46. Find the equation of the curve passing through the point (0,2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5
47. Find the shortest distance between two skew lines
48. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs (i) none(ii) not more than one (iii) more than one , will fuse after 150 days of use.

**PART E**

**V. ANSWER ANY ONE.**

**1X10=10**

49. (a)  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$  if  $f$  is even, Hence evaluate  $\int_{-\pi}^{\pi} \cos x dx$

(b) Prove that 
$$\frac{x+y+2z}{z} \cdot \frac{x}{2x+y+z} \cdot \frac{y}{x+2y+z} = 2(x+y+z)^3$$

50. (a) A manufacturing company makes two products A and B . Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hour for finishing. For fabricating and finishing the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs.8000 on each piece of model A and Rs.12000 on each piece of model B . How many pieces of model A and model B should be manufactured per week to realize a maximum profit? What is the maximum profit?

(b) For what value of 'k'  $f(x) = \begin{cases} kx^2 - 2 & x \leq 0 \\ 4x + 1 & x > 0 \end{cases}$

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