JAIN COLLEGE, Bangalore

Mock Paper - 1 January - 2016

II PUC – Mathematics (35)

Time: 3 Hours 15 Minutes **Max. Marks: 100**

PART A

I. Answer all questions **10 × 1 = 10**

1. Show that $*: R \to R$ given by $(a,b) = a + 4b^2$ is a binary operation.
2. Find the principle value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
3. Define Skew symmetric matrix.
4. If A is an invertible matrix of order 2 and $|A| = 15$ .find $\det(A^{-1})$
5. If $y = \sin^3 x + \cos^6 x$ then find $\frac{dy}{dx}$
6. Find the anti derivative of $(ax+b)^2$ with respect to x
7. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$
8. The Cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ Write its vector form .
9. Define the term "corner points " of LPP.
10. If P(A)=0.8 and P(B)=0.5, $P(B/A) = 0.4$ .Find $P(A \cap B)$

PART B

II. Answer any ten **10 × 2 = 20**

11. Show that the function $f: N \to N$ given by f(1)=f(2)=1 and f(x)=x-1,for every x>2,is onto but not one-one.
12. Prove that $3\sin^{-1} x = \sin^{-1}[3x - 4x^3]$, xϵ[-1/2,1/2]
13. Find the value of $\cos(\sec^{-1} x + \operatorname{cosec}^{-1} x)$
14. Without expansion Prove that $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & a(a+b) \end{vmatrix} = 0$
15. If $y = \log_7(\log x)$ .Prove that $\frac{dy}{dx} = \frac{1}{x \log_7(\log x)}$
16. If $y = \sec^{-1}\left[\frac{1}{2x^2 - 1}\right]$. find $\frac{dy}{dx}$
17. Find the local maximum of the function $g(x) = x^3 - 3x.$
18. Evaluate $\int \log(\sin x).\cot x.dx$
19. Evaluate $\int \tan^{-1} x.dx$
20. Form the differential equation representing the family of curves y=mx,[m is constant].
21. Evaluate $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}(\hat{i} \times \hat{k}) + \hat{k}(\hat{j} \times \hat{i})$

22. Find the area of the parallelogram whose adjacent sides are $\vec{a}=3\hat{i}+\hat{j}+4\hat{k},\vec{b}=\hat{i}-\hat{j}+\hat{k}$
23. Find the vector equation of the line passing through the points (-1,0,2) and (3,4,6).
24. Find the conditional probability of obtaining a sum greater than 9,given that the black die resulted in a 5.

PART C

III. Answer any ten **10 × 3 = 30**

25. Show that the relation R in the set of all integers Z defined by $R=\{(a,b):2\,divides(a-b)\}$ is an equivalence relation.
26. Prove that $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=Tan^{-1}\frac{63}{16}$
27. If A and B are square matrices of same order then Show that $(AB)^{-1}=B^{-1}A^{-1}$
28. If $y^x=x^y=a^b$. find $\frac{dy}{dx}$
29. Prove that the function 'f' given by f(x)=log(cos x) is strictly decreasing on $\left(0,\frac{\pi}{2}\right)$ strictly increasing on $\left(\frac{\pi}{2},\pi\right)$
30. Find the equation of the tangent to the curve $y=x^2-2x+7$ which is parallel to the line $2x-y+9=0$
31. Evaluate $\int\frac{4x+1}{\sqrt{2x^2+x-3}}dx$
32. Evaluate $\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$
33. Find the area of the region bounded by $y^2=9x, x=2, x=4$ and X-axis in I quadrant.
34. Form the differential equation of the family of circles touching y axis at origin.
35. Show that the four points with position vectors $4\hat{i}+8\hat{j}+12\hat{k},2\hat{i}+4\hat{j}+6\hat{k},3\hat{i}+5\hat{j}+4\hat{k}$ and $5\hat{i}+8\hat{j}+5\hat{k}$ are coplanar.
36. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with unit vector along the sum of the vectors $2\hat{i}+4\hat{j}-\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ
37. Find the vector equation of the plane passing through the points R(2,5,-3),S(-2,-3,5),T(5,3,-3).
38. An insurance company insured 2000 scooter drivers,4000 car drivers and 6000 truck drivers. The probability of and accident are 0.01,0.03,0.15 respectively. one of the insured person meets with an accident. What is the probability that he is a scooter driver?

PART D

IV. Answer any six **6 × 5 = 30**

39. Let $y=\{n^2, n\in N\}$ and consider $f:N\to N$ as $f(n)=n^2$,show that f invertible.Find inverse of 'f'.
40. If $A=\begin{bmatrix}1&1&-1\\2&0&3\\3&-1&2\end{bmatrix},B=\begin{bmatrix}1&3\\0&2\\-1&4\end{bmatrix},C=\begin{bmatrix}1&2&3&-4\\2&0&-2&1\end{bmatrix}$ then Prove that (AB)C = A(BC)
41. Solve by matrix method $2x+3y+3z=5, x-2y+z=-4, 3x-y-2z=3.$

42. If $y = Ae^{mx} + Be^{nx}$, Show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

43. A particle moves along the curve $6y = x^3 + 2$ .Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

44. Find the integral of $\sqrt{x^2+a^2}$ with respect to x . and hence evaluate $\int\sqrt{x^2+4x+6}$ .dx.

45. Find the area bounded by the curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$ using integration method

46. Find the equation of a curve passing through the point (0,1) . if the slope of the tangent to the curve at any point (x,y) is equal to the sum of the x-co-ordinate and the product of x-co-ordinate and y- co-ordinate of that point.

47. Derive the equation of a plane in normal form (both in vector and Cartesian form).

48. Find the mean of binomial distribution B(4,1/3)

PART E

V. Answer any one **10 × 1 = 10**

49. a)Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ hence evaluate $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

b)If x,y,z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ Show that 1+xyz=0

50. a) A diet is to contain atleast 80 units of vitamin A and 100 units of minerals. Two foods F1 and F2 are available .food F1 costs Rs 4 per unit and food F2 costs Rs 6 per unit . one unit of food F1 contains 3 units of vitamin A and 4 units of minerals . one unit of food F2 contains 6 units of vitamin A and 3 units of minerals . formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

b) Define a continuity of a function at a point.Find all the points of discontinuity of f defined by $f(x) = |x| - |x+1|$

JAIN COLLEGE, Bangalore
Mock Paper - 2 January - 2016
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Time: 3 Hours 15 Minutes **Max. Marks: 100**

PART A

I. Answer all **10 × 1 = 10**

1. Define objective function.
2. Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.
3. Construct a 2x3 matrix whose elements are given by $a_{ij}=|i-j|$.
4. If A is an invertible matrix of order 2 and $|A|=15$ then find $\det(A^{-1})$.
5. Find the derivative of $\cos(x^2)$ with respect to x.
6. Evaluate $\int(1-x)\sqrt{x}\,dx$.
7. Find the vector joining the points A(2,4,1) and B (-1,3,2).
8. For what value of λ is the vector $\frac{2}{3}\hat{\imath} - \lambda\hat{\jmath} + \frac{2}{3}\hat{k}$ a unit vector.
9. Define optimal solution.
10. The probability of obtaining an even prine number on each die , when a pair of die is rolled is

PART B

II. Answer any 10 **10 × 1 = 10**

11. Define binary operation on a set . verify whether the operation * defined on Z by a∗b=ab+1 is binary or not.
12. Prove that $\tan^{-1}x + \cot^{-1}x = \frac{\Pi}{2}$, $\forall x \in R$.
13. Find the value of $\sin^{-1}(\sin\frac{3\pi}{4})$
14. Using determinants find k if A(1,3) B(0,0) D(K,0) are the vertices of a triangle ABD such that the area of the triangle is 3 sq.units
15. Differentiate $x^{\sin t}$, x>0 w.r.t x
16. Examine the continuity of the function $f(x)=2x^2-1$ at x=3
17. If the radius of a sphere is measured as 7cm with an error of 0.02 m, then find the approximate error in calculating its volume.
18. $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$ dx
19. Evaluate $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$
20. Find the degree of the differential equation $y^{II} + (y^I)^2 + 2y = 0$
21. Find the area of the parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$
22. Find the distance of a point (2,5,-7) from the plane $\vec{r}.(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) = 4$
23. Find the angle between the two planes 2x+y-2z=5 and 3x-6y-2z=7 using vector method.
24. Determine P(E/F). A coin is tossed three times where E: head on 3rd toss, F: heads on first two toss

PART C

III. Answer any 10 **10 × 3 = 30**

25. If * is a binary operation defined on A=N x N, by (a,b)*(c,d)=(a+c, b+d), Prove that * is both commutative and associative. Find the identity element if exist.
26. Find the vaue of x, if $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$
27. Express $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices
28. Verify mean value theorem for $f(x)=x^2-4x-3$ in the interval [a,b] where a=1 and b=4
29. Find the absolute Maximum and absolute minimum value of the function f(x)=sin x+cosx, x∈ $[0,\pi]$
30. Find the equation of the tangent and normal to the curve $y=x^4-6x^3+13x^2-10x+5$ at (0,5)

31. Evaluate $\int_4^9 \frac{\sqrt{x}}{(30-x^{\frac{3}{2}})^2}$ dx

32. Evaluate $\int \frac{x}{(x+1)(x+2)} dx$

33. Find the area of the circle $x^2 + y^2 = a^2$ by integration method.

34. In a bank, principle p increases continuously at the rate of 5% per year. Find the principle in terms of the time t.

35. Find a vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ $where$ $\vec{a}=3\hat{\imath}+2\hat{\jmath}+2\hat{k}$ and $\vec{b}=\hat{\imath}+2\hat{\jmath}-2\hat{k}$

36. Find the area of the triangle ABC where position vectors of A,B and C are $\hat{\imath}-\hat{\jmath}+2\hat{k}$, $2\hat{\jmath}+\hat{k}$ and $\hat{\jmath}+3\hat{k}$ respectively.

37. find the distance between the parallel lines $\vec{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ and $\vec{r}=3\hat{\imath}+3\hat{\jmath}-5\hat{k}+\mu(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$

38. Consider the experiment of tossing two fair coins simultaneously, find the probability that both are head given that at least one of them is a head.

PART D

IV. Answer any six **6 × 5 = 30**

39. Let f:R→R be defined as f(x)=10x + 7. Find the function g:R→R such that $g\circ f=f\circ g=I_R$

40. Let A=$\begin{bmatrix}3 & 7\\2 & 5\end{bmatrix}$ and B=$\begin{bmatrix}6 & 8\\7 & 9\end{bmatrix}$ verify that $(AB)^{-1}=B^{-1}A^{-1}$.

41. Solve the following equations by matrix method x-y+z=4, 2x+y-3z=0,x+y+z=2

42. If $y=\sin^{-1}x$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

43. Sand is pouring from a pipe at the rate of $12cm^3$/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base . How fast is the height of the sand cone increasing when the height is 4cm.

44. Find the integral of $\frac{1}{\sqrt{x^2-a^2}}$ with respect to x and hence evaluate $\int \frac{1}{\sqrt{x^2+2x+2}}$ dx

45. Using integration method, find area of the region bounded by the triangle whose vertices are(1,0), (2,2) and (3,1).

46. Solve the differential equation $ydx-(x+2y^2)dy=0$

47. Derive the condition for the co-planarity of two lines in space both in vector and Cartesian form

48. Find the probability of getting at most two sixes in six throws of a single die.

PART –E

V. Answer any ONE question **1 × 10 =10**

49. (a) Maximise and minimize Z=3x+9y subjected to the constraints.
X+3y≤ 60, x+y≥10, x ≤y, and x≥0 ,y ≥0 graphically.
(b) Show that $\begin{vmatrix}1+a^2-b^2 & 2ab & -2b\\ 2ab & 1-a^2+b^2 & 2a\\ 2b & -2a & 1-a^2-b^2\end{vmatrix} = (1+a^2+b^2)^3$

50. (a) Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & if\ f(x) is\ even\ and \\ 0 & if\ f(x) is\ odd \end{cases}$
And Evaluate $\int_{-1}^{1} sin^5 x cos^4 x\, dx$
(b) Find all the points of discontinuity of f(x), where f is defined by
f(x)=$\begin{cases} x^3-3 & if\ x\geq 2 \\ x^2+1 & if\ x<2 \end{cases}$