| SRI BHAGAWAN MAHAVEER JAIN COLLEGE<br>Vishweshwarapuram, Bangalore.<br><br>**II PUC MOCK QUESTION PAPERS - 2** | **Course:** | II PUC |
| --- | --- | --- |
| | **Subject:** | **Mathematics** |
| | **Max. Marks:** | 100 |
| | **Duration:** | 3:00 Hrs 15 Mins |

## Instructions:

1. **The question paper has five parts namely A, B, C, D and E. Answer all the parts.**
2. **Part A has 10 multiple choice questions, 5 fill in the blanks and Very Short Answer questions of 1 mark each.**
3. **Part A should be answer continuously at one or two pages of Answer sheet an Only first answer is considered for the marks in subsection I and II of Part A**
4. **Use the graph sheet for the question on linear programming in PART E.**

## PART – A

### I. Answer ALL the Multiple choice questions: $10 \times 1 = 10$

1. The identity element for the binary operation is $a*b = a^2 + b^2 \ \forall a,b \in Q$ is
   (A) 0 (B) a (C) b (D) does not exists
2. $\tan^{-1}\frac{x}{y} - \tan^{-1}\left(\frac{x-y}{x+y}\right)$ is equal to
   (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $-\frac{3\pi}{4}$
3. If the matrix A is both symmetric and skew-symmetric, then
   (A) Skew symmetric matrix (B) Symmetric matrix
   (C) A is a square matrix (D) none of these
4. If area of triangle is 35 sq. units with vertices $(2,-6)$, $(5,4)$ and $(k,4)$. Then k is
   (A) 12 (B) – 2 (C) $-12,-2$ (D) $12,-2$
5. The function $f(x) = |x+3|$ is not differentiable at $x =$
   (A) – 3 (B) 1 (C) 2 (D) 3
6. $\int x^2 e^{x^3} dx$ equals
   (A) $\frac{1}{3}e^{x^3} + c$ (B) $\frac{1}{3}e^{x^2} + c$ (C) $\frac{1}{3}e^{x^3} + c$ (D) $\frac{1}{2}e^{x^2} + c$
7. If $\vec{a}$ is a non zero vector of magnitude 'a' and $\lambda\vec{a}$ is unit vector if
   (A) $\lambda = 1$ (B) $\lambda = -1$ (C) $a = |\lambda|$ (D) $a = \frac{1}{\lambda}$
8. The planes $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$
   (A) Perpendicular (B) Parallel
   (C) intersect y-axis (D) passes through $\left(0,0,\frac{5}{4}\right)$
9. The corner points of the bounded region are (0,5), (4,3) & (0, 6). Let $z = 200x + 500y$ be the objective function then the minimum value of z is
   (A) 2300 (B) 4500 (C) 6250 (D) 5000
10. If A and B are two events such that $P(A) \neq 0$ and $P(B|A) = 1$, then
   (A) $A \subset B$ (B) $B \subset A$ (C) $B = \phi$ (D) $A = \phi$

**II Fill in the blanks by choosing the appropriate answer from those given in the brackets**

$(4, 9, 16, 3, \frac{1}{36})$ **$5 \times 1 = 5$**

11. The number of all possible matrices of order 2 x 2 with entry 0 or 1 is________________.
12. The order of the differential equation $\frac{d^3y}{dx^3} + \frac{2d^2y}{dx^2} + \frac{dy}{dx}$ is ____________________.
13. Sum of the intercepts cut off by the plane 6x + 4y + 3z = 12 is__________________.
14. The slope of the tangent to the curve $y = 2x^4 + 4\sin x$ at x = 0 is_________________.
15. The probability of obtaining an odd prime number on each die. When a pair of dice is rolled is_________________________.

**III Answer all the following questions:** **$5 \times 1 = 5$**

16. Define transitive relation.
17. Find the derivative of cos(sinx) w.r.t 'x'.
18. Define Feasible region in a linear programming Problem.
19. Find $\int \frac{1-\cos}{1+\cos x} dx$
20. Define Equal Vectors.

## PART – B

## II Answer any NINE questions: $9 \times 2 = 18$

21. If $f : R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.
22. Simplify $\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$; where $|x| < a$.
23. Find the value odf $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.
24. Show that the points $A(a, b+c)$, $B(b, c+a)$, $C(c, a+b)$ are collinear using determinants.
25. If $x^2 + xy + y^2 = 100$, find $\frac{dy}{dx}$.
26. Find the derivative of $x^x - 2^{\sin x}$ with respect to x.
27. Find a point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.
28. Find $\int e^x \sec x(1 + \tan x) dx$.
29. Find $\int_0^1 \frac{x dx}{1 + x^2}$.
30. Form the differential equation of the family of circles touching the y-axis at origin.
31. If $\vec{a}$ is a unit vector such that $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$. Find $|\vec{x}|$.
32. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, prove that $\vec{a}$ and $\vec{b}$ are perpendicular.
33. The Cartesian equation of a line is $\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$, find the vector equation for the line.

34. Given two independent events A and B such that P(A) = 0.3, P(B) = 0.6. Find
i) P(A and not B) ii) P( neither A nor B).

## PART- C

### III Answer any NINE questions: $9 \times 3 = 27$

35. Show that the relation R in the set Z of integers given by $R = \{(a,b) : 2 \text{ divides } a - b\}$ is an equivalence relation.
36. Solve: $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$.
37. If A and B are symmetric matrices of same order, then show that AB is symmetric if and only if A and B commute, that is AB = BA.
38. If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$, Prove that $\frac{dy}{dx} = \tan\left(\frac{\theta}{2}\right)$
39. Verify mean value theorem for $f(x) = x^2 - 4x - 3$ in the interval $[a,b]$ where $a = 1$ and $b = 4$.
40. Find the interval in which the function $f$ given by $f(x) = 2x^2 - 3x$ is (a) strictly increasing (b) strictly decreasing.
41. Evaluate $\int \frac{4x+1}{\sqrt{2x^2 + x - 3}} dx$.
42. Evaluate $\int \frac{x+1}{x^2 - 4x + 6} dx$.
43. Find the area of the region bounded by the curves $y^2 = 4x$ and $y = 2x$.
44. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x+1}{2-y}$ $(y \neq 2)$.
45. Find the area of triangle having the points $A(1,1,1)$, $B(1,2,3)$ and $C(2,3,1)$ as its vertices.
46. Show that the four points A, B, C and D with position vectors $4\hat{i} + 8\hat{j} + 12\hat{k}$, $2\hat{i} + 4\hat{j} + 6\hat{k}$, $3\hat{i} + 5\hat{j} + 4\hat{k}$ and $5\hat{i} + 8\hat{j} + 5\hat{k}$ respectively are coplanar.
47. Find the equation of the line which passes through the point (1,2,3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$, both in vector form and Cartesian form.
48. If A and B are independent events then show that the probability of occurrence of atleast one of A and B is given by $1 - P(A^{|})P(B^{|})$.

## PART-D

### IV Answer any FIVE questions: $5 \times 5 = 25$

49. Let $f : N \to R$ be defined by $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where S is the range of the function is invertible. Also find the inverse of f.
50. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$ verify that $(AB)^{|} = B^{|}A^{|}$.
51. Solve the following system of equation by matrix method: $2x + 3y + 3z = 5$; $x - 2y + z = -4$; $3x - y - 2z = 3$.
52. If $y = (\tan^{-1} x)^2$ prove that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$.
53. Sand is pouring from a pipe at the rate of 12cm$^3$/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one sixth of the radius of the base. How fast is the height of the sand cone is increasing when the height is 4cm.

54. Find the integral of $\sqrt{a^2 - x^2}$ with respect to x and hence evaluate $\int \sqrt{3-2x-x^2}\,dx$.
55. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$.
56. Solve the diffferential equation $x + y = \frac{dy}{dx} + 5$.
57. Derive the formula to find the shortest distance between two skew lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ in vector form.
58. If a fair coin is tossed 8 times. Find the probability of (i) at least five heads and (ii) at most five heads.

## PART – E

## V Answer the following questions:

59. Maximize $z = 3x + 2y$ subject to the constraints $x + 2y \le 10$; $3x + y \le 15$; $x \ge 0$, $y \ge 0$, by the graphical method.

**OR**

Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & \text{if f is even function} \\ 0 & \text{if f is odd function} \end{cases}$ And hence Evaluate $\int_{-1}^{1} \sin^5 x \cos^4 x dx$

**(6)**

60. Find the value of k so that the function given by $f(x) = \begin{cases} kx+1 & if \quad x \le \pi \\ \text{cosx} & if \quad x > \pi \end{cases}$ is continuous at $x = \pi$.

**OR**

Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \left(1-x^3\right)^2$ **(4)**

* * *