JAIN COLLEGE V V Puram **II PUC Mock Paper 2 :**	**Course:** II year PUC **Subject:** Mathematics **Max. Marks:** 80 **Duration:** 3 hours

1) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*
2) *Part A has 15 Multiple choice questions, 5 Fill in the blanks of 1 mark each.*
3) *Use the graph sheet for question on linear programming in PART E.*

PART-A

I. Answer ALL the Multiple Choice Questions **15 X 1 = 15**

1. Let A = {1, 2, 3} and R be the smallest equivalence relation on A, then R =

a) $R=\{(1,1)\}$ b) $R=\{(1,1),(2,2)\}$

c) $R=\{(1,1),(2,2),(3,3)\}$ d) $R=\{(2,2)(3,3)\}$

2. The domain of $f(x)=\cos^{-1}x$ is

a) $0\le y\le\pi$ b) $-1\le x\le 1$ c) $0<y<\pi$ d) $-1<x<1$

3. The principal value of $\cot^{-1}\left(-\sqrt{3}\right)$ is

a) $-\frac{\pi}{6}$ b) $\frac{\pi}{6}$ c) $\frac{7\pi}{6}$ d) $\frac{5\pi}{6}$

4. The number of all possible matrices of order 3×3 with each entry 0 or 1 is

a) 2^2 b) 2^9 c) 9^2 d) 2^6

5. If A is a square matrix of order 3×3 and $|A|=4$, then $|adj\,A|$.

a) 12 b) 16 c) 64 d) 4

6. Statement 1: $|\sin x|$ is continuous for all $x\in R$.

Statement 2: $\sin x$ and $|x|$ are continuous in R

a) Statement 1 is true and Statement 2 is false.
b) Statement 1 is false and Statement 2 is false
c) Statement 1 is true and Statement 2 is true, Statement 2 is not a correct explanation for Statement 1
d) Statement 1 is true and Statement 2 is true, Statement 2 is a correct explanation for Statement 1

7. If $y=\log_7 2x$, then $\frac{dy}{dx}$ is

a) $\frac{1}{x\ \log_e 7}$ b) $\frac{1}{7x\log_e x}$ c) $\frac{1}{x\log_e 7\log_e x}$ d) $\frac{1}{\log_e 7\log_e x}$

8. Statement 1: The absolute maximum value of the function $2x^3 - 24x$ in the interval $[1,3]$ is 89.

Statement 2: The absolute maximum value of the function can be obtained from the value of the function at critical points and at boundary points.

a) Statement 1 is true and Statement 2 is false.
b) Statement 1 is false and Statement 2 is true
c) Statement 1 is true and Statement 2 is true, Statement 2 is not a correct explanation for Statement
d) Statement 1 is true and Statement 2 is true, Statement 2 is a correct explanation for Statement 1

9. $\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x} dx =$

a) $e-1$ b) $e+1$ c) $\frac{1}{e}+1$ d) $\frac{1}{e}-1$

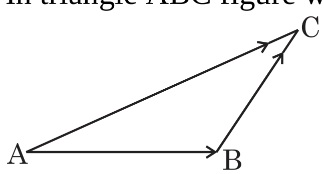
10. A homogeneous differential equation of the from $\frac{dy}{dx} = h\left(\frac{y}{x}\right)$ can be solved by making the substitution.

a) $y = vx$ b) $v = yx$ c) $x = vy$ d) $x = v$

11. The magnitude of the vector $\vec{a} = 6\hat{i} + 2\hat{j} + 3\hat{k}$ is

a) 5 b) 1 c) 12 d) 7

12. In triangle ABC figure which of the following is not true:



a) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$
b) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$
c) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$
d) $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

13. The cartesian equation of the line which passes through the point $(-2,4,-5)$ and is parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is

a) $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$
b) $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$
c) $\frac{x-2}{3} = \frac{y+4}{5} = \frac{z-5}{6}$
d) $\frac{x+3}{-2} = \frac{y-4}{4} = \frac{z+8}{-5}$

14. A die is tossed thrice. The probability of getting an odd number at least once is

a) $\frac{1}{8}$ b) $\frac{3}{8}$ c) $\frac{7}{8}$ d) $\frac{1}{216}$

15. If $P(A) = 0.4, P(B) = 0.8$ and $P(B/A) = 0.6$, then $P(A \cup B) =$

a) 0.24 b) 0.3 c) 0.48 d) 0.96

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket.

[7, 5,1, $\frac{4}{5}$, 0 ,2] **5 X 1 = 5**

16. $\sin\left(\cos^{-1}\frac{3}{5}\right) =$ ______________

17. The number of points at which the function $f(x)=[x]$,where $[x]$ is the greatest integer function is discontinuous in the interval $(-3,3)$ is _______________.

18. $\int_{-\pi}^{\pi} x^4 \sin x \, dx =$ _____________.

19. If a vector makes angles α, β *and* γ with the x-axis, y-axis and z-axis respectively then the value of $(\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$ is________.

20. If A is a subset of B, A and B are two events such that $P(A) \neq 0$. then $P(B/A)$ is_________.

PART-B

Answer any SIX questions. **6 X 2 = 12**

21. Find the area of a triangle whose vertices $(2,7),(1,1) \& (10,8)$ using determinant method

22. Find $\frac{dy}{dx}$, if $ax + by^2 = \cos y$.

23. The length x of a rectangle is decreasing at the rate of $3\, cm/\min$ and the width y is increasing at the rate of $2\, cm/\min$. When $x = 10\, cm$ and $y = 6\, cm$, find the rate of change of the perimeter.

24. Find the local minimum value of the function f given by $f(x) = 3 + |x|,\ x \in R$.

25. Find $\int x^2 \log x \, dx$.

26. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

27. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

28. Find the angle between the pair of lines $\vec{r} = (3\hat{i} + 2j - 4k) + \lambda(\hat{i} + 2j + 2k)$ and $\vec{r} = 5\hat{i} - 2j + \mu(3\hat{i} + 2j + 6k)$.

29. A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even', then prove that E and F are independent events.

PART-C

Answer any SIX questions. **6 X 3 = 18**

30. Show that the relation R in the set of real numbers R defined as $R=\{(a,b): a \le b\}$ is reflexive and transitive but not symmetric.

31. Prove that $\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$.

32. Express the matrix $A=\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

33. If $x = a\cos^3\theta$ *and* $y = a\sin^3\theta$,then prove that $\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$.

34. Find the intervals in which the function f given by $f(x)=x^2-4x+6$ is (i) strictly increasing (ii) strictly decreasing.

35. Evaluate $\int \frac{x}{(x-1)(x-2)}dx$.

36. Show that the position vector of a point P, which divides the line joining the points A and B having position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio $m:n$ is $\frac{m\,\vec{b}+n\,\vec{a}}{m+n}$.

37. Find the shortest distance between the lines $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ and $\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$.

38. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

PART-D

Answer any FOUR questions. **4 X 5 = 20**

39. Verify whether the function, $f: N \to Y$ defined by $f(x)=4x+3$, where $Y=\{y: y=4x+3,\ x\in N\}$ is invertible or not. Write the inverse of $f(x)$ if exist.

40. If $A'=\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

$i)\ (A+B)' = A'+B'$ $ii)\ (A-B)' = A'-B'$

41. Solve the following system of equations by matrix method: $x-y+z=4, 2x+y-3z=0$ and $x+y+z=2$.

42. If $y=3\cos(\log x)+4\sin(\log x)$, show that $x^2y_2+xy_1+y=0$.

43. Find the integral of $\frac{1}{\sqrt{a^2-x^2}}$ with respect to x and hence evaluate $\int\frac{1}{\sqrt{9-25x^2}}dx$.

44. Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ by the method of integration.

45. Find the particular solution of the differential equation $\frac{dy}{dx}+\frac{2xy}{1+x^2}=1$ when $y=0$ and $x=1$.

PART-E

Answer the following question:

46. Prove that $\int_{-a}^{a} f(x)\,dx=\begin{cases}2\int_0^a f(x)dx, & if\ f(x) \text{ is an even function}\\ 0 & ,if\ f(x) \text{ is an odd function}\end{cases}$

and hence evaluate $\int_{-\pi/2}^{\pi/2}\left(x^3+x\cos x+\tan^5 x+1\right)\,dx$

OR

Solve the following problem graphically: Maximize $Z=3x+2y$ Subjected to constraints:

$x+2y\le 10, 3x+y\le 15, x, y\ge 0$ **6**

47. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2-5A+7I=O$, Hence find A^{-1}, where I is the identity matrix of order 2

OR

Find the value of ***k*,** if $f(x)=\begin{cases}kx^2 & if \quad x\le 2\\ 3 & if \quad x>2\end{cases}$ is continuous at $x=2$. **4**