JAIN COLLEGE, J C Road Bangalore
Mock Paper -1, February - 2015

**Time: 3 Hours 15 Minutes** | **II PUC– Mathematics (35)** | **Max. Marks: 100**

## PART - A

**I. Answer all questions** **10 X 1 = 10**

1. Define bijective function.
2. Find the principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$
3. If a matrix has 13 elements, what are the possible orders it can have?
4. Evaluate $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$
5. Define right hand derivative of a function f(x) at x=a.
6. If $y=(2x+1)^3$ then find $\frac{dy}{dx}$
7. Evaluate $\int_2^3 \frac{1}{x}dx$ .
8. Find the vector PQ joining the points P(2,3,0) and Q(-1,-2,-4).
9. Find the intercepts cut off plane 2x+y-z=5.
10. Define constraints of a LPP.

## PART – B

**II. Answer any 10 questions** **10 X 2 = 20**

11. Show that f: $R \to R$ , defined as f(x)= $x^2$ is neither one-one nor onto.
12. P.T $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}, \forall x \in R$ .
13. If $\sin\left[\sin^{-1}\left(\frac{1}{5}\right)+\cos^{-1}x\right]=1$, find x.
14. P.T"If any two rows(columns) of a determinant are interchanged , the sign of determinant changes".
15. Find the derivative of $\cos(\log x+e^x), x>0$ .
16. Differentiate $x^{\sin x}, x>0$ w.r.t x.
17. Find the slope of normal to curve $x=a\cos^3\theta, y=a\sin^3\theta \, at \, \theta=\frac{\pi}{4}$

18. Evaluate $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$

19. Integrate $\frac{\cos\sqrt{x}}{\sqrt{x}}$ $w.r.t\ x$

20. Verify that the function y=acosx+bsinx where $a,b \in R$ is a solution of the differential equation $\frac{d^2y}{dx^2}+y=0$.

21. S.T the vector $\vec{a}=\hat{i}-2j+3k$ , $\vec{b}=-2\hat{i}+3j-4k$ and $\vec{c}=\hat{i}-3j+5k$ are coplanar

22. Find $|\vec{a}|$ *and* $|\vec{b}|$ *, if* $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=8$ *and* $|\vec{a}|=8|\vec{b}|$.

23. Find the angle between pair of lines $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ *and* $\frac{x+1}{1}=\frac{y-4}{1}=\frac{z-5}{2}$.

24. Determine P $\left(\frac{E}{F}\right)$.A coin is tossed 3 times where E: head on third toss and F: heads on first two toss.

## PART- C

**III. Answer any 10 questions** **10 X 3 = 30**

25. Let f: $x \to y$ and g: $y \to z$ be two invertible functions . then show that gof is also invertible with $(gof)^{-1} = f^{-1}og^{-1}$

26. Solve: $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

27. If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$then find the values of x and y.

28. If $x^y=y^x$ then *find* $\frac{dy}{dx}$.

29. P.T the function f is given by f(x)= $\log(\cos x)$ is strictly decreasing on $\left(0,\frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2},\pi\right)$.

30. Find two positive numbers whose sum is 16 and sum of whose cubes is minimum

31. Evaluate $\int \frac{4x+1}{\sqrt{2x^2+x-3}}dx$

32. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx$

33. Find the area of circle $x^2+y^2=a^2$ by integration method.

34. Find the general solution of differential equation $\frac{dy}{dx}=\sin^{-1}x$

35. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$. find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

36. P.T $\left[\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}\right]=2\left[\vec{a}\vec{b}\vec{c}\right]$

37. Find the vector equation of plane passing through the points R(2.5.-3) , S(-2,-3,5) and T(5,3,-3).

38. Probability that A speaks truth is 4/5.A coin is tossed. A reports that a head appears . Find the probability that it is actually head.

## PART – D

**IV. Answer any six questions** **6 X 5 = 30**

39. IF f: $R-\left\{\frac{7}{5}\right\}\rightarrow R-\left\{\frac{3}{5}\right\}$ is given by f(x)= $\frac{3x+4}{5x-7}$ and g: $R-\left\{\frac{3}{5}\right\}\rightarrow R-\left\{\frac{7}{5}\right\}$ defined by g(x)= $\frac{7x+4}{5x-3}$ then S.T fog= $I_A$ and gof= $I_B$.

40. If A= $\begin{bmatrix}1\\-4\\3\end{bmatrix}$,B= $\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$, then verify that $(AB)^{!}=B^{!}A^{!}$

41. The cost of 4kg onion, 3kg wheat and 2kg of rice is Rs.60, the cost of 2kg onion ,4kg wheat and 6kg of rice is Rs.90. the cost of 6kg onion 2kg wheat and 3kg rice is Rs.70 . find the cost of each item per kg by matrix method.

42. If $y=Ae^{mx}+Be^{nx}$ show that: $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny=0$.

43. The radius of a circle is increasing at the rate of 0.7cm/sec what is the rate of increasing of its circumference and also rate of increase of area when r=10cms?

44. Find the integration of $\frac{1}{\sqrt{a^2+x^2}}$ w.r.t x and hence evaluate $\int\sqrt{x^2+2x+5}dx$.

45. Find the area of region in the first quadrant enclosed by x-axis, the line $x=\sqrt{3}y$ and circle $x^2+y^2=4$

46. Find general solution of the differential equation $x\frac{dy}{dx}+2y=x^2\log x$

47. Derive the equation of the plane perpendicular to a given vector and passing through a given point in vector and certesian form.

48. Find the probability distribution of number of doublets in 3 throws of a pair of dice.

## PART – E

**V. Answer any one question** **10 X 1 = 10**

49.

a. Prove that $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ and hence evaluate: $\int_0^4 |x-1|dx$

b. If x,y,z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ then show that 1+xyz=0.

50.

a. Solve the LPP graphically, Minimize and maximize z=x+2y

Subjected to constraints : $x+2y \geq 100, 2x-y \leq 0, 2x+y \leq 200$ , $x, y \geq 0$ .

b. Find all points of discontinuity of f ,where f is defined by

$$f(x) = \begin{cases} |x|+3 & if \quad x \leq -3 \\ -2x & if \quad -3 < x < 3 \\ 6x+2 & if \quad x \geq 3 \end{cases}$$

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Mock Paper -2, February - 2015

**Time: 3 Hours 15 Minutes** **II PUC– Mathematics (35)** **Max. Marks: 100**

## Part – A

### I. Answer all ten 10 X 1 = 10

1. Give an example of a relation which is symmetric and transitive but not reflexive.
2. Write the domain of f(x) = $\sec^{-1}x$.
3. What is the number of possible square matrices of order 3 with each entry 0 or 1?
4. If A is an invertible matrix of order 2 and |A|=15 then find det ($A^{-1}$).
5. If y= $e^{\cos x}$ find dy/dx.
6. Evaluate $\int (ax+b)^2 dx$.
7. Find the value of x and y so that the vectors $2\hat{i}+3\hat{j}$ *and* $x\hat{i}+y\hat{j}$ are equal.
8. Find the distance of the plane 2x – 3y + 4z – 6 = 0 from the origin.
9. Define objective function of a linear programming problem.
10. If P(A) =1/2 , p(B)=1/3 and $P(A\cap B)=1/6$ show that A and B are independent events.

## Part – B

### II. Answer any 10 questions 10 X 2 = 20

11. Define binary operation on a set. Verify whether the operation * defined on Z, by a*b = ab+1 is binary or not?
12. Find the value of $\cot\left[\tan^{-1}(a)+\cot^{-1}(a)\right]$
13. Prove that $\cos^{-1}(-x)=\pi-\cos^{-1}x,\quad x\in[-1,1]$
14. Show that the points A(a,b+c), B(b,c+a) , C(c,a+b) are collinear.
15. If $\sin^2 x+\cos^2 y=1$ find $\frac{dy}{dx}$
16. If $y=\left(\sin^{-1}x\right)^x$ find $\frac{dy}{dx}$
17. Find the approximate change in volume V of a cube of side 'x' meters caused by increasing the side by 1%.
18. Evaluate $\int x^2 e^{x^3} dx$

19. Find $\int_0^{\frac{\pi}{2}} \cos 2x dx$

20. Form the differential equation representing the family of curves y=mx where m is arbitrary constant.

21. Find the angle between 2 vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$

22. Find the area of the parallelogram whose adjacent sides are the vectors $3\hat{i}+\hat{j}+4\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$

23. Find the distance of the point (2,3,-5) from the plane $\vec{r}.(\hat{i}+2\hat{j}-2\hat{k})=9$

24. The random variable x has a probability distribution P(x) of the following from where k is some number $p(x)=\begin{cases} k & if \ x=0 \\ 2k & if \ x=1 \\ 3k & if \ x=2 \\ 0 & otherwise \end{cases}$ Determine the value of k.

## PART – C

**III. Answer any 10 questions** **10 X 3 = 30**

25. Show that if $f:A\to B$ and $g:B\to C$ Are one – one then $g\circ f:A\to C$ Is one-one

26. Prove that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

27. Find the values of x,y and z in matrix $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix}=\begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

28. If $y=\tan^{-1}\left[\frac{\sin x}{1+\cos x}\right]$ prove that $\frac{dy}{dx}=\frac{1}{2}$

29. Verify Rolle's theorem for the function $f(x)=x^2+2x-8$ , $x\in[-4,2]$

30. Find the point on the curve $y=x^3-11x+5$ At which the tangent is $y=x-11$

31. Evaluate $\int \frac{(1+\log x)^2}{x}dx$

32. Evaluate $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$

33. Find the area bounded by parabola $y^2=4x$ . the line y=2x

34. Form the differential equation representing the family of curves y=asin(x+b). Where a and b arbitrary constant.

35. If $\vec{a}=2\hat{i}-3j+4k$ , $\vec{b}=\hat{i}+2j-3k$ and $\vec{c}=3\hat{i}+4j-k$ Then find $\vec{a}.(\vec{b}X\vec{c})$ and $(\vec{a}X\vec{b}).\vec{c}$ .

36. If $\vec{a}=-4\hat{i}-6j-\lambda k$ , $\vec{b}=-\hat{i}+4j+3k$ and $\vec{c}=-8\hat{i}-j+3k$ are coplanar . Find $\lambda$ .

37. Find the distance between parallel lines $\vec{r}=\hat{i}+2j-4k+m(2\hat{i}+3j+6k)$ and $\vec{r}=3\hat{i}+3j-5k+n(2\hat{i}+3j+6k)$

38. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of aces.

## Part – D

**IV. Answer any 6 questions** **6 X 5 = 30**

39. Consider $f:R\rightarrow R$ defined by $f(x)=4x+3$ show that f is invertible. Find the inverse of f.

40. If A= $\begin{bmatrix}1 & 2 & 3\\ -4 & 5 & 6\\ 7 & 8 & 0\end{bmatrix}$, B= $\begin{bmatrix}2 & 3 & 4\\ 5 & -3 & 0\\ 4 & 5 & -3\end{bmatrix}$ and C= $\begin{bmatrix}2 & 3 & 1\\ 4 & 5 & 6\\ -1 & 2 & 3\end{bmatrix}$ Prove that A(B+C)=AB+AC

41. Solve the system of linear equations by matrix method
$x-y+2z=7, 3x+4y-5z=-5 \ and \ 2x-y+3z=12$

42. If $y=3\cos(\log x)+4\sin(\log x)$ show that $x^2y_2+xy_1+y=0$

43. A man of height 2 meters walk at a uniform speed of 5km /hr away from a lamp post which is 6meters height. Find the rate at which the length of his shadow increases.

44. Find the integral of $\sqrt{x^2+a^2}$ With respect to 'x' and evaluate $\int\sqrt{4x^2+9}dx$

45. Find the area of the circle $4x^2+4y^2=9$ which is interior to the parabola $x^2=4y$

46. Solve the differential equation $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$

47. Derive equation of line passing through 2 given points both in vector and cartesian form.

48. Find the mean of binomial distribution B(4,1/3)

## Part - E

**V. Answer any 1 question** **10 X 1 = 10**

49. (a) Prove that $\int_0^{2a} f(x)dx=\begin{cases}2\int_0^a f(x)dx & if \quad f(2a-x)=f(x)\\ 0 & if \quad f(2a-x)=-f(x)\end{cases}$ and hence evaluate $\int_0^{\pi}\frac{xdx}{a^2\cos^2 x+b^2\sin^2 x}$

(b) Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \left(1-x^3\right)^2$

50. (a) One kind of cake requires 200g of flour and 25g of fat and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1kg of fat assuming that there is no shortage of other ingredients used in making the cakes. Formulate the LPP and solve it graphically .

(b) Find the value of k so that the function 'f' given by $f(x) = \begin{cases} kx+1 & if \quad x \le \pi \\ \cos x & if \quad x > \pi \end{cases}$ is continuous at $x = \pi$ .