JAIN COLLEGE, Bangalore
Mock Paper - 1 January - 2019
II PUC – Mathematics (35)

Time: 3 Hours 15 Minutes **Max. Marks: 100**

PART A

I. Answer all questions: **10 × 1 = 10**

1. State a relation which is symmetric but not transitive and reflexive.
2. Write the domain of $\sec^{-1} X$.
3. If $X+\begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix}=\begin{bmatrix} 2 & 4 \\ 5 & 0 \end{bmatrix}$, find matrix X.
4. If A is invertible matrix of order 2 find det $|A^{-1}|$.
5. Find $\frac{dy}{dx}$, $if\ y=\log(\cos e^{x})$
6. Find the anti derivative of $\cot^2 X$ with respect to x.
7. For what value of λ, the vector $\vec{a}=2i-3j+k\ and\ \vec{b}=i+j-2k$ are perpendicular to each other.
8. Find the direction ratios of the line $\frac{x-1}{2}=3y=\frac{3z+3}{4}$
9. Define feasible region.
10. A fair die is rolled . consider the event E={ 1,3,5} and F={2,3}, find p(E/F).

PART B

II. Answer any ten : **10 × 2 = 20**

11. Let A be on the set R-{1} such that a*b=a+b-ab, check if the operation is associative or not.
12. Find $\cos^{-1}(\cos\frac{7\pi}{6})$
13. write the simplest form of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x\neq 0.$
14. For the matrix A=$\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ find the value of a & b such that $A^2+aA+bI=0$
15. Find $\frac{dy}{dx}, y=\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$
16. Differentiate 3x+3y=siny with respect to x.
17. Determine the interval in which the function $f(x)=x^3+5x^2-1$ is increasing and decreasing.
18. $\int\frac{dx}{\sin^2 x\cos^2 x}$
19. $\int\frac{dx}{x-\sqrt{x}}$
20. Form the differential equation representing the family of curves Y=mX , m is arbitrary constant.
21. Show that the vector $i+j+k$ is equally inclined to the positive OX,OY, and OZ axis.
22. Find the area of the triangle with vertices A(1,1,2) B(2,3,5) and c(1,5,5) by vector method.

23. Find the Cartesian equation of a line which passes through the point (-2,4,-5) and parallel to the line

$$\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$$

24. The random variable X has a probability distribution

p(x) = $\left\{\begin{array}{ll} k, & if\ x=0 \\ 2k, & if\ x=1 \\ 3k, & if\ x=2 \\ 0 & otherwise \end{array}\right\}$ determine the value of K.

PART C

III. Answer any ten: 10 × 3 = 30

25. Show that the relation R on the set of Z given by $R=\{(x,y): 2 divides (x-y)\}$ is a equivalent relation.

26. Solve for x: $\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}\left(\frac{x}{3}\right)=\frac{\pi}{4}$

27. Using the elementary operation , find the inverse of the matrix $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

28. Verify the mean value theorem for the equation $f(x)=x^2-4x-3$ in the interval [1,4].

29. If $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$ prove that $\frac{dy}{dx}=\tan\frac{\theta}{2}$.

30. Find the equation of the tangent and normal to the curve $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point $(\sqrt{2}a, b)$.

31. $\int\frac{dx}{x^2-6x+13}$

32. $\int\sin 3x\cos 4x\,dx$

33. Find the area bounded by the curve $Y^2=4X$ and the line x=3.

34. Solve the differential equation $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$

35. Show that the four points with position vectors

$4i+8j+12k$, $2i+4j+6k$, $3i+5j+4k$ and $5i+8j+5k$ are coplanar.

36. For any three vectors $\vec{a},\vec{b}$ and $\vec{c}$,prove that $\left[\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}\right]=2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

37. Find the shortest distance between the two lines

$\vec{r}=(i+2j+3k)+\lambda(i-3j+2k)$ and $\vec{r}=(4i+5j+6k)+\mu(2i+3j+k)$

38. An urn contains 4 white and 6 red balls. Four balls are drawn at a random (without replacement). Find the probability distribution of the number of white balls.

PART D

IV. Answer any six : 6 × 5 = 30

39. Let f:N→R be defined by $f(x)=4X^2+12X+15$ .Show that f is invertible and also find the inverse.

40. Verify (A+B)C=AC+BC , if $A=\begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix} B=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} and\, c=\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$

41. Solve by matrix method
    a. x-y=3 ,2x+3y+4z=17, y+2z=7.

42. If y= 3cos(logX)+4sin(logX) show that $X^2\, Y_2$ +X Y_1 +y=0 .

43. The volume of a cube is increasing at a rate of 9 centimeter cube per second. How fast is the surface area increasing, when the length of the edge is 10 cms.

44. Find the integral of $\frac{1}{\sqrt{a^2-x^2}}$ with respect to x and hence evaluate $\int \frac{dx}{\sqrt{7-6x-x^2}}$

45. Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by the method of integration.

46. Find the general solution of the differential equation $\frac{dy}{dx}+y\cot x=4x\,\mathrm{cos}ecx$

47. Derive the equation of the plane perpendicular to the given vector and passing through the given points both in vector and Cartesian form.

48. If a fair coin is tossed 8 times. Find the probability of (i) at least 5 heads (ii) at most five heads.

PART E

V. Answer any one : 10 × 1 = 10

49.

a. Prove that : $\int_0^{\frac{\pi}{2}} \log(\sin x)dx = -\frac{\pi}{2}\log 2$ (6)

b. Prove by property: $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$ (4)

50.

a. Minimize and Maximize , solve graphically the linear equations Z= X+2Y (6)

X+2Y$\geq$ 100

2X-Y $\geq$ 0

2X+Y$\leq$ 200 X,Y≥ 0

b. Find the points of discontinuity of the function f defined by (4)

$$f(x)=\begin{cases} \frac{x}{|x|} & if\ x<0 \\ -1 & if\ x>0 \end{cases}$$

JAIN COLLEGE, Bangalore
Mock Paper - 2 January - 2019
II PUC – Mathematics (35)

Time: 3 Hours 15 Minutes **Max. Marks: 100**

PART A

I. Answer all : **10 × 1 = 10**

1. Operation * is defined by a*b=a, is *a Binary operation on set of integers?
2. Write the values of x for which $2tan^{-1}x = cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$ holds.
3. Construct a 2x2 matrix A= [a_{ij}] whose elements are given by $\frac{1}{2}|-3i + j|$.
4. If A is a square matrix, $|A| = 8$, then find the value $|AA'|$.
5. If $y = e^{\log(\cos x)^{1/2}}$, find dy/dx.
6. Evaluate $\int sec\, x\,(\sec x + \tan x)dx$.
7. Find λ when the projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4.
8. Find the direction cosine of a line $\frac{x-3}{4} = \frac{y+2}{5} = \frac{z}{2}$.
9. Define objective function.
10. If P(A)=3/5, P(B)=1/5, find P(A∩B), if A & B are independent events.

PART B

II. Answer any 10 : **10 × 1 = 10**

11. Prove that the greatest integer function f: R→R, defined by f(x)=[x], is neither One-one nor onto.
12. Write $tan^{-1}\left[\frac{1}{\sqrt{x^2-1}}\right], |x| > 1$in the simplest form.
13. Prove that $\cos^{-1}(-x) = \pi - \cos^{-1}x$,x€[0,π]
14. Without expansion Prove $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0.$
15. Differentiate $e^{-x^2}.\sin(logx)$ wrt x.
16. Verify Rolle's theorem for the function $f(x)=x^3-4x$ in the interval [-2,2].
17. Find the maximum & minimum value of the function $f(x) = 3x^4-8x^3+12x^2-48x+25$ in [0,3].
18. Find $\int \log x\, dx$.
19. $\int \frac{1}{1+\tan x}\, dx$
20. Find the order and degree of the Differential equation. $y'' + sin(y')^2 + 2y = 0$.
21. Find $[\vec{a}\quad \vec{b}\quad \vec{c}]$ if $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}, \vec{b} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$.
22. Find the magnitude , if 2 vectors $\vec{a}$ and $\vec{b}$ have the same magnitude and angle between them is 60^0 and their scalar product is ½.
23. Find the cartesian equation of the following plane $\vec{r}.(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}) = 1$.
24. Find the conditional probability of obtaining a sum greater than 9 when a black and a red die is rolled, given that black die resulted in a 5.

PART C

III. Answer any 10 : **10 × 3 = 30**

25. If * is a Binary operation on Q, $a * b = \frac{3ab}{5}$ check whether it is associative & commutative. Find its identity element if it exists.

26. Prove that. $3\cos^{-1}x = \cos^{-1}(4x^3-3x)$, $x \in [\frac{1}{2},1]$.

27. Express $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ as sum of symmetric and skew symmetric matrices.

28. If $y = sec^{-1}\left[\frac{1}{2x^2-1}\right]$ find dy/dx.

29. Using differentials find the approximate value of $(26)^{1/3}$.

30. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is maximum.

31. Evaluate as s limit of sum $\int_1^2 x^2$ dx.

32. Evaluate $\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}\,dx$.

33. Find the area of the circle $x^2+y^2 = a^2$ by integration method.

34. Find the particular solution of the differential equation $\frac{dy}{dx} = -4xy^2$ given that y=1 and x = 0.

35. If $\vec{a}, \vec{b}$ & $\vec{c}$ are unit vector such that $\vec{a} + \vec{b} + \vec{c} = 0$, find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.

36. The scalar product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with unit vector along the sum of the vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to one. Find λ.

37. Find the vector equation of the plane passing through the points R(2,5,-3); S(-2,-3,5) and T(5,3,-3).

38. In a meeting 70% of the members are in favour and 30% oppose the proposal. A member is selected at random and we take x=0 if he opposed and x=1 if he is in favour. Find E(x) and V(x).

PART D

IV. Answer any six : 6 × 5 = 30

39. If f:R⟶R and $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$ then show that the function is invertible and find its inverse?

40. If $A = \begin{bmatrix}2 & 0 & 1\\0 & -3 & 0\\0 & 0 & 4\end{bmatrix}$, verify $A^3-3A^2-10A+24I=0$.

41. The sum of three numbers is 6. Three times the third number added to the second number gives 11. By adding first and the third number, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

42. If $y = e^{a\sin^{-1}x}$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$.

43. A particle is moving along the curve $6y = x^3+2$. Find the points on the curve at which the y-co-ordinate is changing 8 times fast as the x- co-ordinate.

44. Find the integral of $\frac{1}{a^2+x^2}$ w.r.t x and hence evaluate $\int \frac{3x^2}{x^6+1}dx$.

45. Using integration method, find the area of the region bounded by the triangle whose vertices are (1,0); (2,2) and (3,1).

46. Find the solution of the differential equation, $x\frac{dy}{dx} - y + \sin\left(\frac{y}{x}\right) = 0$.

47. Derive the equation of a line which passes through two given points. Both vector and Cartesian form.

48. Find the probability distribution of number of doublets in 3 throws of a pair of dice, find the mean and variance.

PART –E

V. Answer any ONE question: 1 × 10 =10

49. a) Prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ and hence evaluate $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{tanx}}dx$.

b) Find the value of 'a', if the function $f(x) = \begin{cases}2x-1, & x < 2\\ a, & x = 2\\ x+1, & x > 2\end{cases}$ is continuous at x = 2.

50. a) A furniture dealer deals in only 2 items tables and chair. He has Rs.50000 to invest and has a storage space of atmost 60 pieces. A table costs Rs.2500 and a chair Rs.500. He estimates that from the sale of one table, he can make a profit of Rs.250 and that from one chair is Rs.75. How many tables and chairs should he buy for the available money so as to maximize his profit assuming that he can sell all the items which he buys. Solve the problem graphically.

b) Using properties of determinant, if x,y,z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then 1+xyz=0.